Rationalising the denominator

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\dfrac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\dfrac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Example 1 Rationalise $\dfrac{1}{\sqrt{3}}$

$\dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$	**1** Multiply the numerator and denominator by $\sqrt{3}$
$= \dfrac{1 \times \sqrt{3}}{\sqrt{9}}$	**2** Use $\sqrt{9} = 3$
$= \dfrac{\sqrt{3}}{3}$	

Example 2 Rationalise and simplify $\dfrac{\sqrt{2}}{\sqrt{12}}$

$\dfrac{\sqrt{2}}{\sqrt{12}} = \dfrac{\sqrt{2}}{\sqrt{12}} \times \dfrac{\sqrt{12}}{\sqrt{12}}$	**1** Multiply the numerator and denominator by $\sqrt{12}$
$= \dfrac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$	**2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number
$= \dfrac{2\sqrt{2}\sqrt{3}}{12}$	**3** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **4** Use $\sqrt{4} = 2$
$= \dfrac{\sqrt{2}\sqrt{3}}{6}$	**5** Simplify the fraction: $\dfrac{2}{12}$ simplifies to $\dfrac{1}{6}$

Example 3 Rationalise and simplify $\dfrac{3}{2+\sqrt{5}}$

$\dfrac{3}{2+\sqrt{5}} = \dfrac{3}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \dfrac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	**2** Expand the brackets
$= \dfrac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \dfrac{6-3\sqrt{5}}{-1}$	**4** Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1
$= 3\sqrt{5}-6$	

Practice questions

1 Rationalise and simplify, if possible.

    a $\dfrac{1}{\sqrt{5}}$ **b** $\dfrac{1}{\sqrt{11}}$

    c $\dfrac{2}{\sqrt{7}}$ **d** $\dfrac{2}{\sqrt{8}}$

    e $\dfrac{2}{\sqrt{2}}$ **f** $\dfrac{5}{\sqrt{5}}$

    g $\dfrac{\sqrt{8}}{\sqrt{24}}$ **h** $\dfrac{\sqrt{5}}{\sqrt{45}}$

2 Rationalise and simplify.

    a $\dfrac{1}{3-\sqrt{5}}$ **b** $\dfrac{2}{4+\sqrt{3}}$ **c** $\dfrac{6}{5-\sqrt{2}}$

3 Rationalise and simplify, if possible.

    a $\dfrac{1}{\sqrt{9}-\sqrt{8}}$ **b** $\dfrac{1}{\sqrt{x}-\sqrt{y}}$

Answers

1   **a** $\dfrac{\sqrt{5}}{5}$ **b** $\dfrac{\sqrt{11}}{11}$

    c $\dfrac{2\sqrt{7}}{7}$ **d** $\dfrac{\sqrt{2}}{2}$

    e $\sqrt{2}$ **f** $\sqrt{5}$

    g $\dfrac{\sqrt{3}}{3}$ **h** $\dfrac{1}{3}$

2   **a** $\dfrac{3+\sqrt{5}}{4}$ **b** $\dfrac{2(4-\sqrt{3})}{13}$ **c** $\dfrac{6(5+\sqrt{2})}{23}$

3   **a** $3+2\sqrt{2}$ **b** $\dfrac{\sqrt{x}+\sqrt{y}}{x-y}$